Supplementary Materials

Real-Time Birth-to-Annihilation Dynamics of Dissipative Kerr Cavity Soliton

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S1. Theoretical model

To theoretically understand the nonlinear dynamics of the Kerr fiber cavity, a mean-field Lugiato-Lefever equation (LLE) is employed to describe the behavior of the light propagation, i.e.,

$$\frac{\partial E}{\partial z} = -i\frac{\delta_0}{L}E - \frac{\alpha}{2L}E - i\frac{\beta_2}{2}\frac{\partial^2 E}{\partial t^2} + i\gamma \mid E \mid^2 E + \frac{\theta}{L}E_{in}. \qquad (S1)$$

where E is the slowly-varying envelope of the intracavity light field and E_{in} is the field amplitude of the driving field. δ_0 represents the linear phase cavity detuning that is defined as the difference between the linear cavity roundtrip phase shift and the phase of the closest cavity resonance.[1] β_2 and γ are the group-velocity dispersion (anomalous in this case) and nonlinear coefficient of the single-mode fiber (SMF), respectively. L is the length of the Kerr fiber cavity. α is the total cavity loss, while θ accounts for the amplitude transmittance of the driving pump. In the experiment, the key parameters include $\beta_2 \approx -20\ \mathrm{ps^2/km}$, $\gamma \approx 1.5\ \mathrm{W^{-1}\,km^{-1}}$, $L \approx 53$ m, $\alpha \approx 0.16$, $\theta \approx \sqrt{0.05}$.

To explore the regimes of the parameter space in the LLE, we study the cavity response using a function $P = f(P_{in})$ that is written as

$$\frac{P}{P_{in}} = \frac{\theta^2}{(\delta_0 - \gamma PL)^2 + \alpha^2/4}, \qquad (S2)$$

where P and P_{in} are the optical powers of the intracavity and driving fields. Eq. (S2) shows a S-shaped curve and results in multiple stability regimes when $\delta_0 > \sqrt{3}\alpha/2$, and the upper ($P_+$) and lower ($P_-$) branches of homogeneous solutions, i.e.,

$$P_\pm = \frac{4\delta_0 \pm \sqrt{4\delta_0^2 - 3\alpha^2}}{6\gamma L}, \qquad (S3)$$

Eq. (S3) defines the bistable regime that latently supports the cavity soliton (CS).[1] In order to further analyze the spontaneous generation of the CS, the Turing-type modulation instability (MI) of the mean-field LLE[2] can be characterized using the gain spectrum $G(\omega)$ as well as its most unstable frequency ω_{MI}, i.e.,

$$G(\omega) = -\frac{\alpha}{2L} + \sqrt{(\gamma P_{in})^2 - \left(\frac{\beta_2\omega^2}{2} + 2\gamma P_{in} - \frac{\delta_0}{L}\right)^2}, \quad (S4a)$$

$$\omega_{MI} = \sqrt{\frac{2}{\beta_2}\left(\frac{\delta_0}{L} - 2\gamma P_{in}\right)},\ G(\omega_{MI}) = -\frac{\alpha}{2L} + \gamma P_{in}. \quad (S4b)$$

Considering the inequalities of $G(\omega_{MI}) \geq 0$ and $\omega_{MI} \geq 0$ that give rise to the MI, two critical power levels that designate the MI regime in the parameter space (δ_0, P_{in}) i.e.,

$$P_{th} = \frac{\alpha}{2\gamma L}, \quad P_{\omega} = \frac{\delta_0}{2\gamma L}. \quad (S5)$$

S2. Numerical simulation

The dynamics of the CS are numerically investigated by solving Eq. (S1) using the split-step method with a fourth-order Runge-Kutta algorithm. To facilitate the parameter settings that render versatile dynamics, the coefficients used in Eq. (S1) can be rewritten in dimensionless forms by means of the transformations, i.e.,

$$Z \to \frac{\alpha z}{2L},\ T \to t\sqrt{\frac{\alpha}{\beta_2 L}},\ E' \to E\sqrt{\frac{2\gamma L}{\alpha}}, \quad (S6a)$$

$$\Delta \to \frac{2\delta_0}{\alpha},\ S \to E_{in}\sqrt{\frac{8\gamma L\theta^2}{\alpha^3}}. \quad (S6b)$$

Thus, Eq. (S1) is revised as

$$\frac{\partial E'}{\partial Z} = -i\Delta E' - E' - i\frac{\partial^2 E'}{\partial T^2} + i\mid E\mid^2 E + \mathrm{S}. \quad (S7)$$

For a driving pump of $P_{in} \approx 6$ W that is consistent with the experiment, it gives rise to $S \approx 7.1$. A scan of cavity detuning δ_0 follows the Δ variation from 6 to 15.

S3. Data processing

The experimental data recorded by the real-time oscilloscope are firstly segmented according to the roundtrip time of the Kerr fiber cavity, and formatted as a two-dimensional matrix *M*×*N*, in which each column (*M*) designates spectral information and each row (*N*) represents the roundtrip number. For the time-stretched signal, a coordinate transform is applied in terms of

$\lambda = t/D_2$, where t and λ represent the retarded time and wavelength, respectively. D_2 is the amount of dispersion provided by the five dispersion compensating modules, i.e., -10.2 ns/nm in this case. To improve the signal-to-noise ratio, a lowpass filtering is applied.

Reference

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